

# Quantum corrections to critical phenomena in gravitational collapse

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We investigate conformally coupled quantum matter fields on spherically symmetric, continuously self-similar backgrounds. By exploiting the symmetry associated with the self-similarity the general structure of the renormalized quantum stress-energy tensor can be derived. As an immediate application we consider a combination of classical, and quantum perturbations about exactly critical collapse. Generalizing the standard argument which explains the scaling law for black hole mass,  $M \propto |\eta - \eta^*|^\beta$ , we demonstrate the existence of a quantum mass gap when the classical critical exponent satisfies  $\beta \geq 0.5$ . When  $\beta < 0.5$  our argument is inconclusive; the semi-classical approximation breaks down in the spacetime region of interest. [S0556-2821(98)02414-X]

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## I. INTRODUCTION

Choptuik [1] demonstrated that, for suitably chosen initial data, black holes of arbitrarily small mass can form in the gravitational collapse of a massless scalar field. Specifically, if the strength of the initial data is characterized by some parameter  $\eta$ , say, then there exists a critical value  $\eta^*$  such that the corresponding solutions to the Einstein-scalar field equations are divided into three classes:

(i) *Sub-critical solutions* have  $\eta < \eta^*$ ; the collapsing matter eventually disperses leaving behind flat space.

(ii) *Critical solutions* have  $\eta = \eta^*$ ; they exhibit self-similar echoing in the neighborhood of a central singularity. The same echoing solution develops independent of the shape of the initial data.

(iii) *Super-critical solutions* have  $\eta > \eta^*$ ; the scalar field collapses to form a black hole. The masses of black holes which form in marginally super-critical evolutions obey a scaling law such that

$$M \propto |\eta - \eta^*|^\beta \quad (1.1)$$

where  $\beta \approx 0.37$  is independent of the initial data.

Since Choptuik's initial discovery, critical point behavior has been studied in a variety of models for gravitational collapse [2–5]. Whenever black hole formation turns on at infinitesimal mass, precisely critical evolutions exhibit some form of self-similarity and the black-hole mass scales as in Eq. (1.1) with a model dependent exponent.

The properties of black holes are radically changed by quantum field theory. Since the work of Hawking [6] it is well known that black holes formed by gravitational collapse will radiate particles via quantum processes precisely as a black body with a temperature, the Hawking temperature, proportional to the surface gravity of the black hole. For a Schwarzschild black hole the Hawking temperature,  $T_H$ , is given by

$$kT_H = \frac{\hbar c^3}{8\pi GM} \quad (1.2)$$

where  $M$  is the mass of the black hole. Consequently, the black hole radiates energy at a rate proportional to  $M^{-2}$ . It follows that a black hole of mass  $M$  evaporates away by the Hawking process in approximately  $10^{-26}(M/1 \text{ g})^3$  seconds; black holes of very low mass, such as those formed in marginally super-critical collapse, evaporate away almost instantaneously.

It would be of great interest to determine how the classical picture of marginally supercritical collapse is modified by quantum gravity; unfortunately this is beyond current techniques. A more modest program is to examine quantum fields in critical spacetimes, and from this study to infer the semi-classical corrections to the classical evolutions. In this paper we undertake such an investigation. We focus attention on models of gravitational collapse in which black-hole formation turns on at infinitesimal mass, *and* the critical solution exhibits continuous self-similarity:<sup>1</sup> perfect fluid collapse and a class of Brans-Dicke models belong in this category. The continuous self-similarity allows us to infer a great deal about the renormalized quantum stress-energy tensor for conformally coupled fields in a precisely critical spacetime. Moreover, generalizing the classical, perturbative treatments of Koike *et al.* [7], Maison [8], and Gundlach [9] to include semi-classical corrections, we can infer the presence of a quantum mass-gap at the threshold of black hole formation when  $\beta > 0.5$ . (For perfect fluids with pressure proportional to energy density, i.e.  $p = k\rho$ ,  $\beta$  increases monotonically from about 0.106 when  $k=0$  to 0.820 when  $k=0.899$  passing through  $\beta=0.5$  at  $k \approx 0.53$ .) Our argument is quite robust, requiring minimal assumptions about semi-classical corrections to general relativity. It is worth noting that a mass-gap which originates with quantum effects will not be universal in general. This observation is a direct consequence of the non-locality of the renormalized stress-energy tensor which carries information about the initial data which led to the collapse.

<sup>1</sup>By continuous self-similarity, we mean that there exists a vector field  $\xi$  such that Eq. (2.4) is satisfied.

We may contrast our approach with that of two related sets of work. First, Ayal and Piran [10] have made a detailed numerical study of scalar-field collapse in general relativity including a quantum stress-energy tensor, inspired by two dimensional considerations, as a source. There are two differences with our approach: (1) As Ayal and Piran deal with scalar field collapse the critical spacetime in their case contains only discrete self-similarity and not continuous self-similarity as we have assumed here. (2) The quantum stress-energy tensor used by Ayal and Piran is not exactly conserved; by contrast ours arises from a renormalized effective action and so by construction is conserved. Second, Bose *et al.* [11,12] have studied semi-classical effects in gravitational collapse in the framework of two-dimensional dilaton theories. They have shown that quantum effects lead to a mass-gap at the threshold of black hole formation in this theory. Unfortunately, it is not clear whether the critical solution in their model exhibits any form of self-similarity, so it is difficult to directly compare with the present work.

The paper is organized as follows. In Sec. II we introduce self-similarity in the context of spherically symmetric spacetimes. The purpose is to highlight those features which are important in the subsequent discussion of the renormalized stress-energy tensor (RSET). This discussion is presented in Sec. III A which reviews the properties of the conformal transformation law for the RSET of a conformally invariant field. This law contains anomalous terms arising from the trace anomaly. Self-similarity allows the metric of the collapse spacetime to be written in a conformally stationary form. The conformal transformation law for the RSET then allows time-dependence of the RSET to be derived in these spacetimes. In Sec. III C we show how this information can be incorporated into the standard classical, perturbative treatment to include semi-classical corrections. From this analysis we infer that, when semi-classical effects are accounted for, there is a mass gap at the threshold of black hole formation when the classical critical exponent exceeds  $\beta > 0.5$ . We finish with a brief discussion of the results.

## II. SELF-SIMILARITY AND SPHERICAL SYMMETRY

It is convenient to use a retarded coordinate  $u$ , and to write the spherical line element as

$$ds^2 = e^{-2u}[-G(u, \zeta)du^2 - 2H(u, \zeta)dud\zeta + \zeta^2 d\Omega^2], \quad (2.1)$$

where  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ . Notice that the radius of the two-spheres is given by

$$r(u, \zeta) = \zeta e^{-u}. \quad (2.2)$$

Since these coordinates are unfamiliar it is worth elucidating a couple of simple, but important, points. The origin of the coordinate  $\zeta$  coincides with the symmetry origin, i.e.  $r(u, 0) = 0$ . Surfaces of constant  $\zeta$  are timelike in a neighborhood of the origin; generally this neighborhood does not extend over the entire patch covered by the coordinates  $(u, \zeta)$ . This is clearly demonstrated by an example. In Minkowski spacetime the metric functions are  $G(u, \zeta) = 1 - 2\zeta e^{-u}$  and

$H(u, \zeta) = 1$ , so that  $\zeta$  changes from timelike to spacelike when  $\zeta = e^u/2$ . Nevertheless, the metric is manifestly regular across this hypersurface. Finally, we note that  $r \rightarrow \infty$  as  $u \rightarrow -\infty$  on surfaces of constant  $\zeta$ , while  $r \rightarrow 0$  as  $u \rightarrow \infty$  along the same surfaces.

The spacetimes of interest below evolve from regular initial data, and develop a singularity at  $r=0$  as a result of gravitational collapse of some matter field. We normalize  $u$  so that the singularity is located at infinite coordinate time, and that the proper time as measured by an observer at the origin is exponentially related to  $u$ , that is  $(\tau - \tau_s) \propto e^{-u}$  where  $\tau_s$  is the proper time at the singularity.

Black-hole formation may be inferred from the existence of an apparent horizon which expands to meet the event horizon at late times (assuming that cosmic censorship holds). Surfaces of constant radius change from timelike to spacelike at the apparent horizon in spherical spacetimes. Thus, the normal to a surface of constant radius is null at the apparent horizon, and the equation for an apparent horizon is

$$g^{\alpha\beta} \nabla_\alpha r \nabla_\beta r = (G + 2\zeta H)H^{-2} = 0. \quad (2.3)$$

Self-similar spacetimes are characterized by the existence of a vector field  $\xi$  such that

$$\mathcal{L}_\xi g = -2g, \quad (2.4)$$

where  $g$  is the metric tensor. The above coordinates are well adapted to discuss self-similarity since the line element for a spherically symmetric, self-similar spacetime can always be written as in Eq. (2.1) with

$$G(u, \zeta) = G_{ss}(\zeta), \quad (2.5)$$

$$H(u, \zeta) = H_{ss}(\zeta), \quad (2.6)$$

and  $\xi = \partial/\partial u$ . Explicitly, we have

$$ds^2 = e^{-2u}[-G_{ss}(\zeta)du^2 - 2H_{ss}(\zeta)dud\zeta + \zeta^2 d\Omega^2], \quad (2.7)$$

so that  $g_{\mu\nu} = e^{-2u} \bar{g}_{\mu\nu}$  where  $\xi = \partial/\partial u$  is a Killing vector for the metric  $\bar{g}_{\mu\nu}$ .

In some studies of phase transitions in gravitational collapse, continuous self-similarity is observed in near critical evolutions when black hole formation turns on at infinitesimal mass. This is schematically depicted in Fig. 1 where the spacetime diagram represents the collapse of critical ( $\eta = \eta^*$ ) initial data. The shaded region indicates the asymptotic approach to self similarity in the central region; in *precisely* critical evolutions this region extends all the way to the singularity.

## III. SEMI-CLASSICAL THEORY OF CRITICAL COLLAPSE

The explicit computation of the renormalized stress-energy tensor for quantum fields is beyond current techniques except in certain exceptional circumstances with high symmetry. Nonetheless, significant progress can be made by

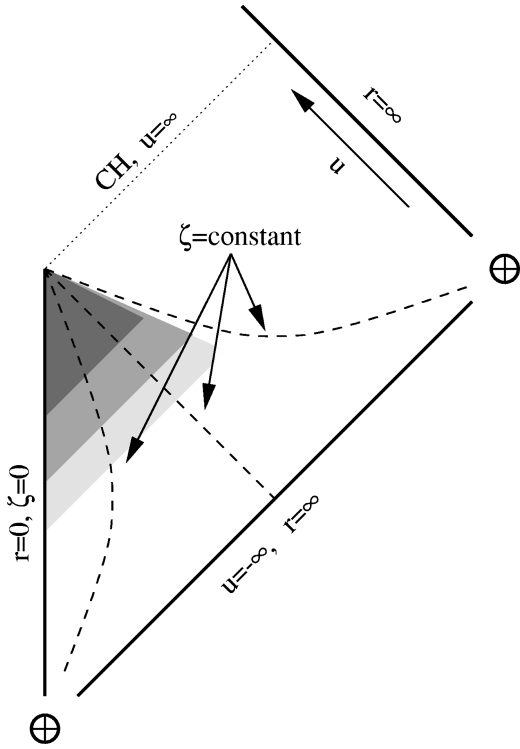


FIG. 1. Schematic representation of the spacetime of critical collapse. Surfaces of constant  $\zeta$  are indicated as dashed lines. The retarded time coordinate goes from  $u = -\infty$  at past null infinity to  $u = \infty$  at the Cauchy horizon indicated by CH; a singularity forms at  $r = \zeta = 0$ ,  $u = \infty$ . The shaded region indicates the asymptotic approach to self-similarity near the singularity.

exploiting the self-similarity of critical spacetimes and a powerful tool which has been developed by Page *et al.* [13,14].

### A. Renormalized stress-energy tensor

At the quantum level it is well known that renormalization breaks the conformal invariance of a classically conformally invariant theory. This is manifested in the existence of the trace anomaly. As a result the renormalized stress-energy tensor does not simply scale under conformal transformation but also acquires geometrical corrections. Quite generally, Page has shown that the RSET for conformally coupled fields in a spacetime  $(\mathcal{M}, g)$  can be obtained from the RSET in the conformally related spacetime  $(\mathcal{M}, \bar{g})$ , where  $g_{\mu\nu} = e^{-2\omega} \bar{g}_{\mu\nu}$ , by the following transformation rule:

$$\begin{aligned} \langle T_{\mu}^{\nu} \rangle = & e^{4\omega} \overline{\langle T_{\mu}^{\nu} \rangle} + 8\alpha e^{4\omega} [(\omega \bar{C}_{\alpha\mu\beta}^{\nu})^{:\alpha\beta} \\ & + \frac{1}{2} \omega \bar{R}^{\alpha\beta} \bar{C}_{\alpha\mu\beta}^{\nu} - \beta [(2H_{\mu}^{\nu} - 4R^{\alpha}_{\beta} C_{\alpha\mu}^{\beta\nu}) \\ & - e^{4\omega} (2\bar{H}_{\mu}^{\nu} - 4\bar{R}^{\alpha}_{\beta} \bar{C}_{\alpha\mu}^{\beta\nu})] - \frac{1}{6} \gamma [I_{\mu}^{\nu} - e^{4\omega} \bar{I}_{\mu}^{\nu}]]. \end{aligned} \quad (3.1)$$

Here a colon denotes covariant differentiation with the natural connection for the metric  $\bar{g}_{\mu\nu}$ :

$$H^{\mu\nu} = -R^{\mu}_{\alpha} R^{\alpha\nu} + \frac{2}{3} R R^{\mu\nu} + (\frac{1}{2} R_{\alpha\beta} R^{\alpha\beta} - \frac{1}{4} R^2) g^{\mu\nu}, \quad (3.2)$$

and

$$\begin{aligned} I^{\mu\nu} = & \frac{1}{\sqrt{g}} \frac{\delta}{\delta g_{\mu\nu}} \int \sqrt{g} d^4 x R^2 \\ = & 2R^{:\mu\nu} - 2R R^{\mu\nu} + (\frac{1}{2} R^2 - 2R_{;\alpha}^{;\alpha}) g^{\mu\nu}. \end{aligned} \quad (3.3)$$

Here,  $\langle T_{\mu}^{\nu} \rangle$  and  $\overline{\langle T_{\mu}^{\nu} \rangle}$  denote the RSET in some state on  $(\mathcal{M}, g)$  and in the conformally related state on  $(\mathcal{M}, \bar{g})$ . The coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  depend on the spin of the field; if  $h_s$  denotes the number of helicity states for fields of spin  $s$ , then

$$\alpha = [12h_0 + 18h_{1/2} + 72h_1] / (2^9 45 \pi^2) \quad (3.4a)$$

$$\beta = [-4h_0 - 11h_{1/2} - 124h_1] / (2^9 45 \pi^2) \quad (3.4b)$$

$$\gamma = [8h_0 + 12h_{1/2} + (48 \text{ or } -72)h_1] / (2^9 45 \pi^2). \quad (3.4c)$$

The ambiguity in the coefficient of  $h_1$  arises from the choice of renormalization method but is irrelevant to our discussion.

For our purposes, it is convenient to rewrite the first term using the identity

$$\begin{aligned} (\omega \bar{C}_{\alpha\mu\beta}^{\nu})^{:\alpha\beta} + \frac{1}{2} \omega \bar{R}^{\alpha\beta} \bar{C}_{\alpha\mu\beta}^{\nu} = & \omega \bar{B}_{\mu}^{\nu} + \omega^{:\beta} \bar{C}_{\alpha\mu\beta}^{\nu:\alpha} \\ & + \omega^{:\alpha} \bar{C}_{\alpha\mu\beta}^{\nu:\beta} + \omega^{:\alpha\beta} \bar{C}_{\alpha\mu\beta}^{\nu} \end{aligned} \quad (3.5)$$

where the Bach tensor  $B^{\mu\nu}$  is defined as

$$\begin{aligned} B^{\mu\nu} = & \frac{1}{\sqrt{g}} \frac{\delta}{\delta g_{\mu\nu}} \int \sqrt{g} d^4 x \frac{1}{4} C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} \\ = & C_{\alpha}^{\mu}{}_{\beta}{}^{\nu:\alpha\beta} + \frac{1}{2} R^{\alpha\beta} C_{\alpha}^{\mu}{}_{\beta}{}^{\nu}. \end{aligned} \quad (3.6)$$

Furthermore, since [15]

$$R^{\alpha\beta}_{\gamma\delta} = e^{4\omega} [\bar{R}^{\alpha\beta}_{\gamma\delta} + \text{terms involving } \omega_{;\rho}] \quad (3.7)$$

we find

$$\begin{aligned} \langle T_{\mu}^{\nu} \rangle = & e^{4\omega} \overline{\langle T_{\mu}^{\nu} \rangle} + 8\alpha \omega e^{4\omega} \bar{B}_{\mu}^{\nu} \\ & + e^{4\omega} [\text{terms involving } \omega_{;\rho}]. \end{aligned} \quad (3.8)$$

There is an ambiguity in the RSET relating to the  $\omega$  contribution, that is, the *logarithm* of the conformal factor in Eq. (3.1) that we must now discuss. There is an arbitrary renormalization scale hidden in the logarithm so that a constant conformal transformation, which corresponds simply to a change in length scale, changes the RSET by the addition of a multiple of the Bach tensor. (Since the Bach tensor arises from a conformally invariant action it is traceless, so this ambiguity does not effect the trace anomaly.) In the next subsection we will see that this merely corresponds to a

choice of renormalization point for the coupling coefficients in a generalized Einstein action.

A direct application of Eq. (3.8) to the self-similar spacetimes of Eq. (2.7) with  $e^{-\omega} = e^{-u}$  determines the  $u$  dependence of the RSET in the physical spacetime. Schematically, we can write

$$\langle T_{\mu}^{\nu} \rangle = e^{4u} \overline{\langle T_{\mu}^{\nu} \rangle}(\zeta) + 8\alpha u e^{4u} \bar{B}_{\mu}^{\nu}(\zeta) + e^{4u} \bar{S}_{\mu}^{\nu}(\zeta), \quad (3.9)$$

where  $\bar{S}_{\mu}^{\nu}(\zeta)$  denotes a tensor constructed from the geometry of  $(\mathcal{M}, \bar{g})$  and  $\omega_{;\mu} = \delta_{\mu}^u$  which cannot depend on  $u$  since  $\xi = \partial/\partial u$  is a Killing vector of  $(\mathcal{M}, \bar{g})$ . The state dependence is carried by the RSET  $\overline{\langle T_{\mu}^{\nu} \rangle}(\zeta)$  computed in the conformal spacetime; it is independent of  $u$  since  $\xi$  is a Killing vector in this spacetime, and we expect the quantum states of interest to respect this symmetry.

### B. Semi-classical equations

In general, quantum field theory in curved spacetime is only renormalizable (at one-loop) when viewed as part of a general theory of the gravitational field with a low energy effective action of the form

$$I = \int_{\mathcal{M}} \left( \frac{1}{16\pi G} (R - 2\Lambda) + \frac{a}{4} C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} + b R^2 \right). \quad (3.10)$$

The coupling constants  $\Lambda$ ,  $a$  and  $b$  of this effective theory must be measured. Therefore the ambiguity associated with the regularization scale is a manifestation of our lack of knowledge of physics at Planck scales—the boundary of validity of any effective theory based on an expansion of the gravitational action in powers of curvature. The generalized (semi-classical) Einstein equations can be written as

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G [T^{\mu\nu} + \epsilon (\langle T^{\mu\nu} \rangle + 2a B^{\mu\nu} + 2b I^{\mu\nu})] \quad (3.11)$$

where  $\epsilon$  is a counting parameter which is unity if semi-classical effects are included and zero otherwise.

The classical, critical solution corresponds to a self-similar solution to these equations with  $\epsilon=0$  and  $\Lambda=0$ . We wish to consider perturbations to such solutions which originate with small deviations from critical initial data in the presence of quantum matter. A non-zero cosmological constant would change the value of  $\eta$  at the critical point, but the asymptotic solution should be unchanged provided  $1/\sqrt{\Lambda}$  is larger than the initial matter configuration. For this reason we assume that  $\Lambda$  remains zero. Thus, we look for solutions of these generalized equations of the form

$$G(\zeta, u) = G_{ss}(\zeta) - (\eta - \eta^*) g_c(\zeta) e^{\omega_c u} + \epsilon g_q(\zeta) e^{\omega_q u} \quad (3.12)$$

$$H(\zeta, u) = H_{ss}(\zeta) - (\eta - \eta^*) h_c(\zeta) e^{\omega_c u} + \epsilon h_q(\zeta) e^{\omega_q u}. \quad (3.13)$$

It is unnecessary to consider changes in  $g_{\theta\theta}$  since they can always be removed by a first order coordinate transformation. The value of  $\omega_c$  is determined by solving a boundary value problem for the classical perturbations of the self-similar solution; this has been done by several authors [8,7,9]. However,  $\omega_q=2$  is easily determined by computing the Einstein tensor to linear order in  $\hbar$  for the line-element in Eq. (2.1) [with  $G(\zeta, u)$  and  $H(\zeta, u)$  determined by Eqs. (3.12) and (3.13) respectively], and comparing the  $u$  dependence with that of the RSET in Eq. (3.11) as determined by Eq. (3.9).

### C. Modified mass scaling

We can now consider the modified scaling relation for black hole mass. In the self-similar spacetimes corresponding to the critical point of gravitational collapse  $G_{ss} + 2H_{ss}\zeta > 0$  everywhere, i.e., there is no apparent horizon.

Substituting the perturbed quantities into Eq. (2.3), the apparent horizon is located at  $(\zeta_h, u_h)$  such that

$$\begin{aligned} F(\zeta_h, u_h) &= G_{ss} + 2H_{ss}\zeta_h - (\eta - \eta^*)(g_c + 2h_c\zeta_h) e^{\omega_c u_h} \\ &\quad + \epsilon(g_q + 2h_q\zeta_h) e^{\omega_q u_h} \\ &= 0. \end{aligned} \quad (3.14)$$

Now, the radius of the apparent horizon is related to  $(u_h, \zeta_h)$  by  $R_h = e^{-u_h} \zeta_h$  so that Eq. (3.14) can be rewritten as

$$\begin{aligned} F(R_h, \zeta_h) &= G_{ss} + 2H_{ss}\zeta_h \\ &\quad - (\eta - \eta^*)(g_c + 2h_c\zeta_h) \zeta_h^{\omega_c} R_h^{-\omega_c} \\ &\quad + \epsilon(g_q + 2h_q\zeta_h) \zeta_h^{\omega_q} R_h^{-\omega_q} \\ &= 0. \end{aligned} \quad (3.15)$$

The classical limit ( $\epsilon=0$ ) has been explored by other authors who have argued that the observed scaling relation for black hole mass is determined by solving Eq. (3.15) for  $R_h$  in this limit [7,8]. Thus, one arrives at the relation

$$R_h \propto (\eta - \eta^*)^{1/\omega_c}. \quad (3.16)$$

For perfect fluids with pressure proportional to energy density, i.e.,  $p = k\rho$ , the classical parameter  $\omega_c$  decreases monotonically from about 9.46 when  $k=0$  to 1.22 when  $k = 0.899$  passing through  $\omega_c = 2$  at  $k \approx 0.53$  [7,8].

As the mass of the black hole which forms in marginally super-critical collapse approaches the Planck mass, quantum effects will become significant. Moreover, it is reasonable to expect that quantum matter will compete with gravitational collapse eventually averting formation of a black hole for some  $\eta_q$  sufficiently close to  $\eta^*$ . The conclusions that can be drawn from our analysis depend strongly on the relative magnitudes of  $\omega_c$  and  $\omega_q$ , therefore we break the discussion into two cases.

(i) When  $\omega_c < \omega_q$  the function  $F(R_h, \zeta_h)$  is represented schematically in Fig. 2. It is approximately constant during the self-similar phase of the evolution. For sufficiently large  $\eta$ , classical gravitational collapse takes hold and a black hole

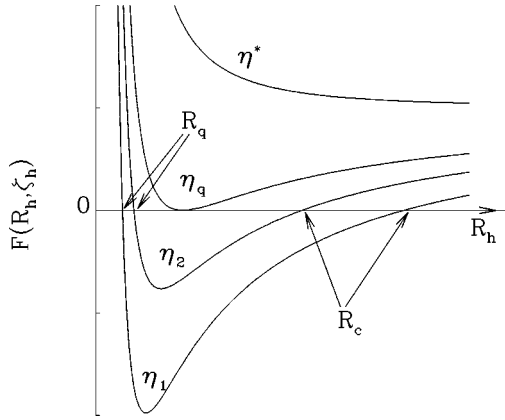


FIG. 2. The horizon location is determined by the roots of the function  $F(R_h, \zeta_h)$  in Eq. (3.15). We show here a schematic representation for several values of  $\eta$  which determine deviations from classically critical initial data, and  $\omega_c < \omega_q$ . For sufficiently large  $\eta$  classical collapse takes hold and a black hole forms at  $R_c$ . The function has a minimum, however, and another root  $R_q$  exists. As  $\eta$  is tuned to a critical value  $\eta_q$  the two roots coincide. When  $\eta < \eta_q$  no black hole forms. Thus a mass gap exists at the threshold of black hole formation in semi-classical collapse.

forms at  $R_c$ . Note that the function has a minimum at smaller  $R_h$ , and a second root at  $R_q$ . As  $\eta$  decreases  $R_c \rightarrow R_q$  until the roots coincide at some critical value of the parameter  $\eta_q$ . When  $\eta < \eta_q$ , no black hole forms. Thus, we can infer a mass-gap at the threshold of black hole formation in semi-classical collapse.

(ii) When  $\omega_c \geq \omega_q$  we can say less about the critical point. Figure 3 shows  $F(R_h, \zeta_h)$  in this circumstance for the two cases  $\epsilon=0$  and  $\epsilon=1$ . As  $\eta \rightarrow \eta^*$  semi-classical effects have a significant effect causing the radius of the apparent horizon to be reduced compared to the purely classical result; there is only a single root of Eq. (3.15). Once again, there is a critical value  $\eta_q$  of the parameter which marks the point when the apparent horizon radius corresponds to the boundary at which curvatures reach Planck scales and we can no longer trust semi-classical calculations. Quantum gravity, or at least a better approximation to it, is needed to properly determine the critical point behavior.

#### IV. DISCUSSION

The conclusion arrived at here is not rigorous. We do not fully understand quantum gravity, or how to fully incorpo-

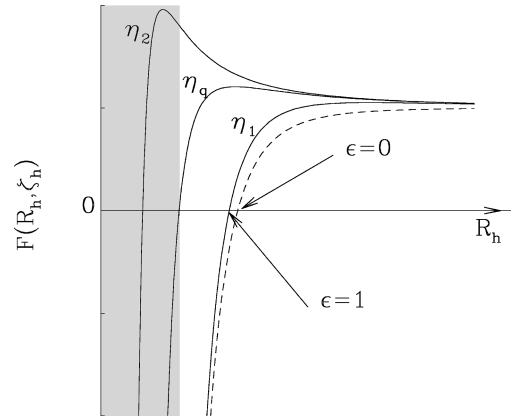


FIG. 3. The horizon location is determined by the roots of the function  $F(R_h, \zeta_h)$  in Eq. (3.15). When  $\omega_c \geq \omega_q$  and  $\eta \rightarrow \eta^*$  the function has a local maximum, however the classical terms always dominate as  $R_h \rightarrow 0$ . The dashed line is  $F(R_h, \zeta_h)$  in the absence of quantum corrections when  $\eta = \eta_1$ . By assumption, quantum corrections decrease the size of the black hole as indicated by the slight decrease in the root when  $\epsilon=1$  and  $\eta = \eta_1$ . When  $\eta = \eta_q$  the black-hole horizon lies at the boundary of Planckian curvature and we must appeal to quantum gravity to understand the quantum corrections to the near critical evolutions.

rate semi-classical effects into gravity. Nonetheless we have been able to make some progress in understanding semi-classical effects in critical spacetimes by studying the structure of the renormalized stress-energy tensor for conformally coupled fields in the critical background spacetime. By modifying the perturbative arguments which are used to obtain the critical exponent observed in classical collapse we have been able to infer a mass gap at the threshold of black hole formation in semi-classical theory. This conclusion relies heavily on the assumption that quantum matter tends to oppose black-hole formation. The validity of this assumption can not be addressed without a complete calculation of the renormalized stress-energy tensor in the dynamical spacetimes of near critical collapse. Such a computation would be very difficult requiring the development of new techniques.

#### ACKNOWLEDGMENTS

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